



ECE 344

Microwave Fundamentals

Assistant Professor
Dr. Gehan Sami

Example 1(2.2 in the book)

$Z_0=100, z_L = 0.4+j0.7$
($Z_L=40+j70$)

$l=.3\lambda$

Find $\Gamma_L, \Gamma(-l), Z_{in},$
SWR, Return loss

From smith chart

We can find:

From reflection coefficient E scale

$|\Gamma_L| = .59, \theta = 104^\circ$

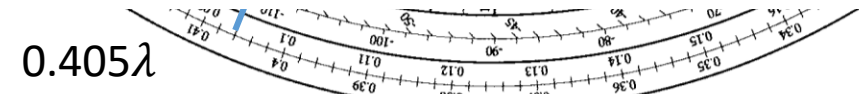
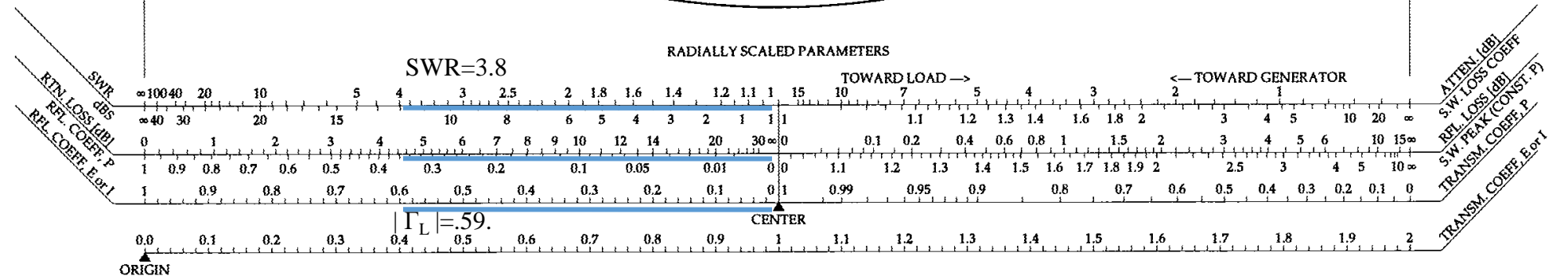
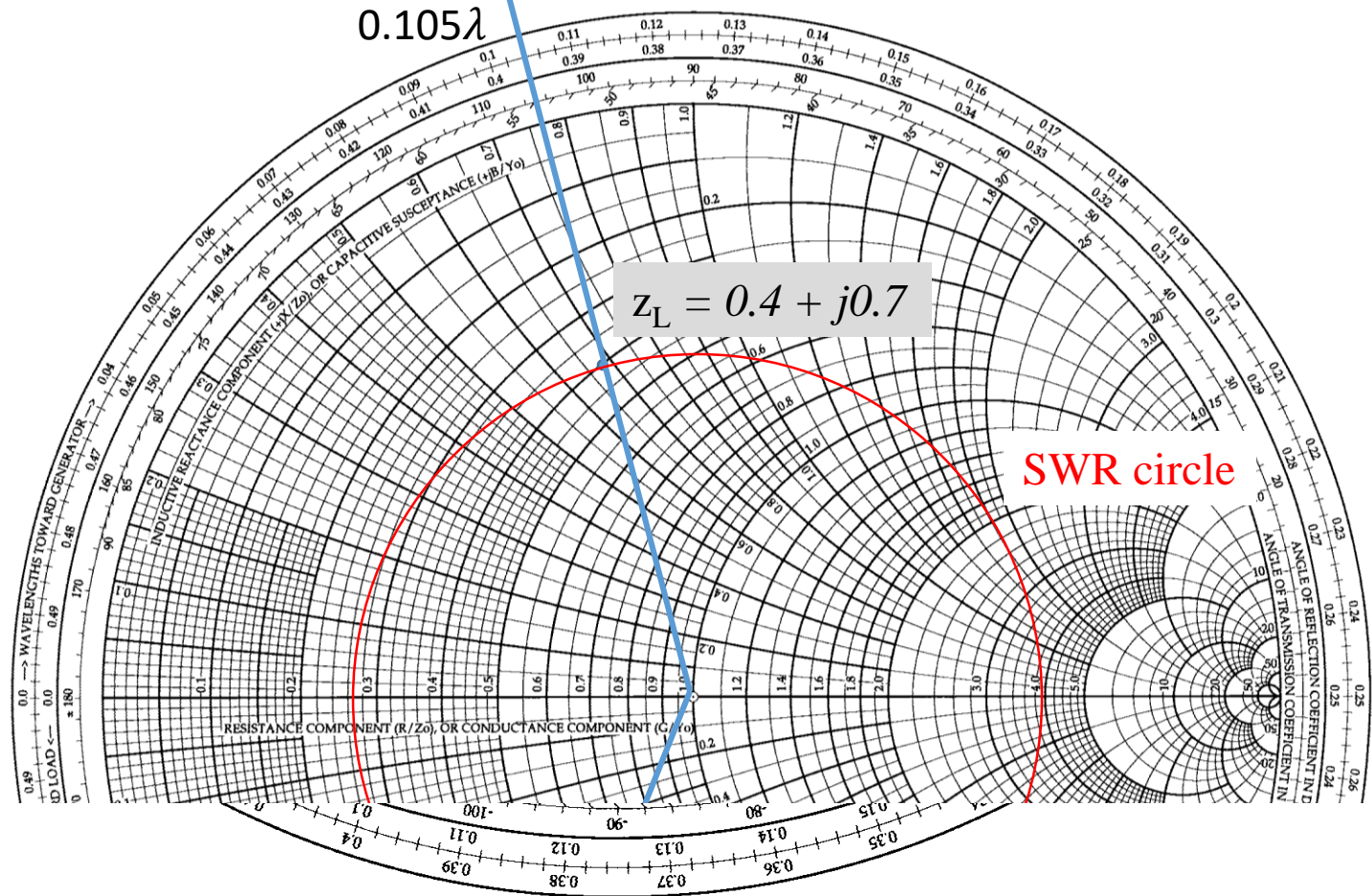
From SWR scale

SWR=3.8

From return loss scale, return

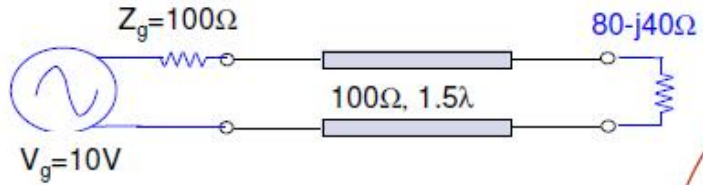
loss=4.6dB

$\Gamma(-l) = .59 e^{-j1}$



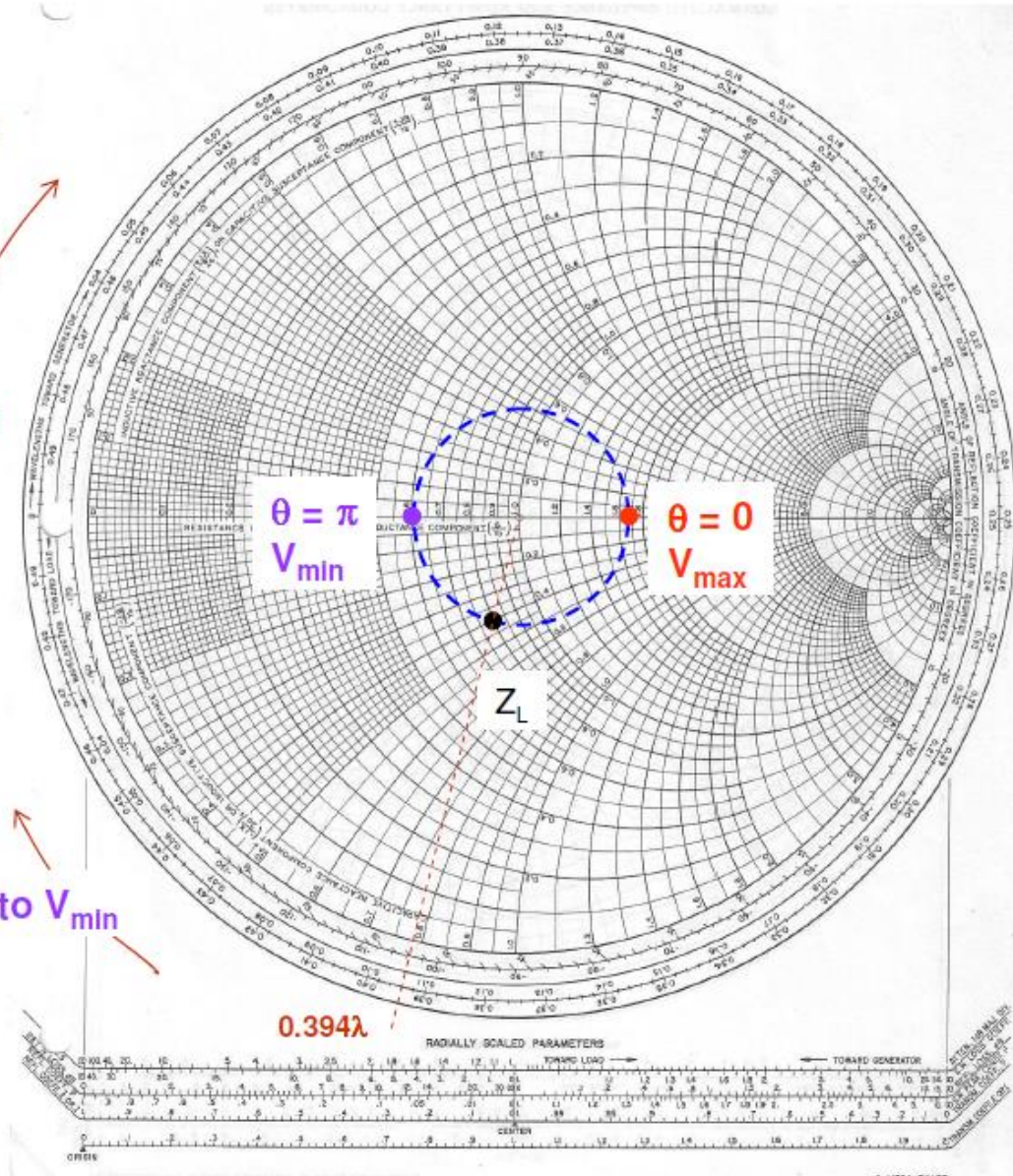
V_{\max} & V_{\min}

Example 2



0.356λ to V_{\max}

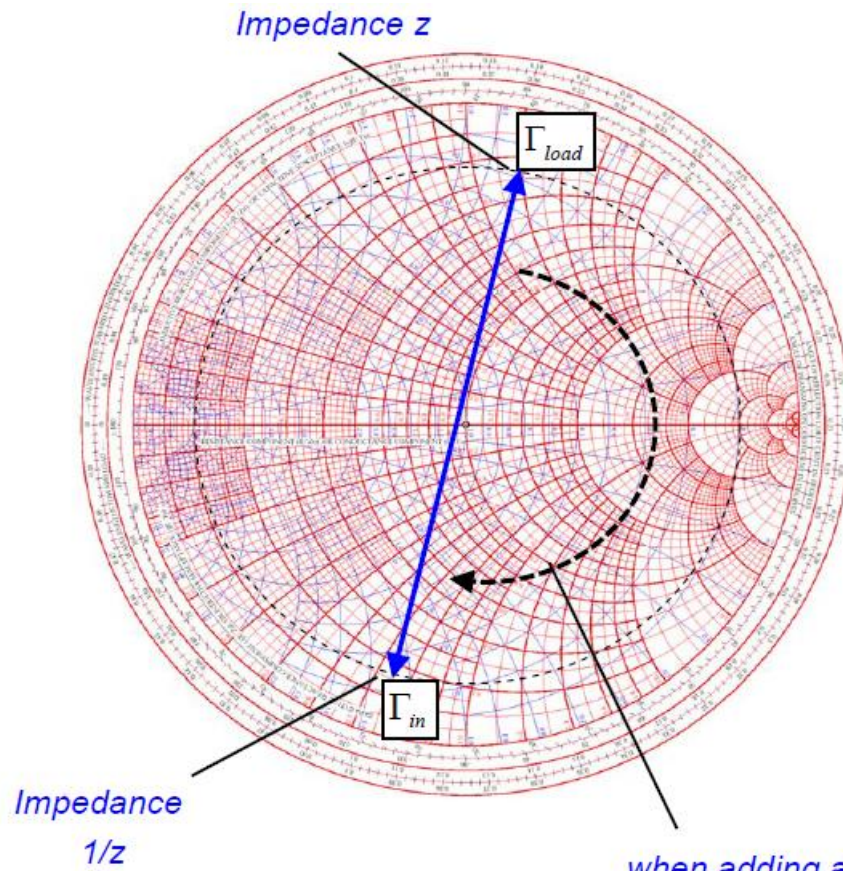
0.106λ to V_{\min}



Effect of moving TWG $\lambda/4$

$$z_{in}(l = \lambda/4) = \frac{1}{z_L} = y_L \quad \text{where } z_{in} = \frac{Z_{in}}{z_0}, \quad z_L = \frac{Z_L}{z_0}, \quad y_L = \frac{Y_L}{y_0}$$

the length $\lambda/4 \rightarrow$ moving π on smith chart \rightarrow gives us admittance of the load



A transmission line of length

$$l = \lambda/4$$

transforms a load reflection Γ_{load} to its input as

$$\Gamma_{in} = \Gamma_{load} e^{-j2\beta l} = \Gamma_{load} e^{-j\pi} = \underline{-\Gamma_{load}}$$

Thus a normalized load impedance z is transformed into $1/z$.

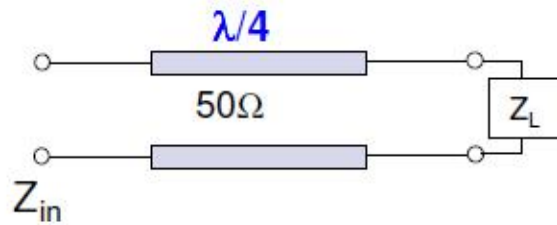
In particular, a short circuit at one end is transformed into an open circuit at the other. This is a particular property of the $\lambda/4$ transformers.

when adding a transmission line to some terminating impedance we move clockwise through the Smith-Chart.

Effect of moving TWG $\lambda/4$

Example 3

e.g. $Z_L = 50 + j 25 \Omega$



$$Z_{in} = 50(0.8 - j 0.4) \Omega$$

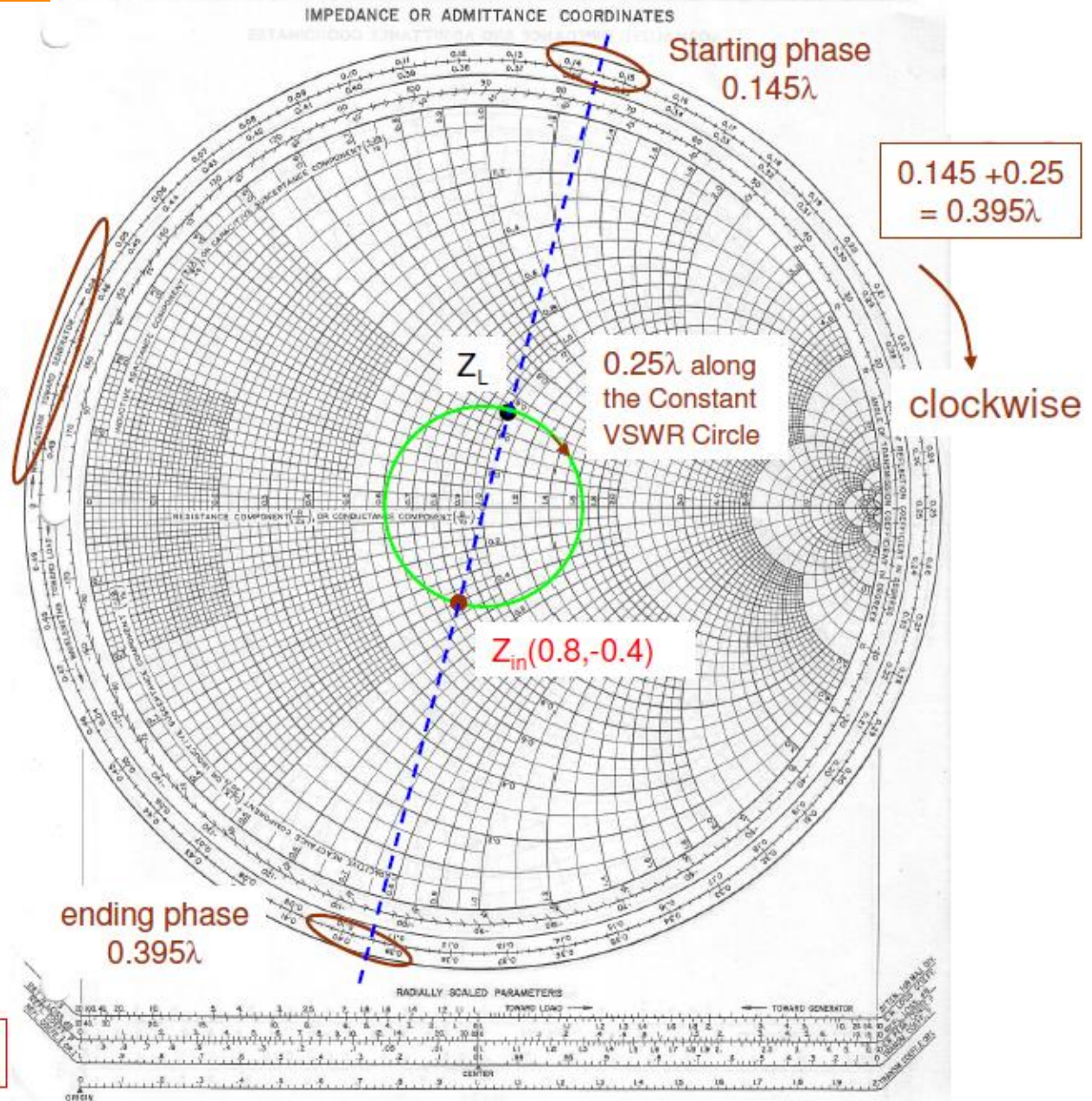
$$Z_o = \sqrt{Z_{in} Z_L}$$

$$1 = \sqrt{\bar{Z}_{in} \bar{Z}_L}$$

$$\bar{Z}_{in} = \frac{1}{Z_L} = \bar{Y}_L$$

$$Y_L = 0.02(0.8 - j 0.4) \Omega^{-1}$$

Convert $Z \leftrightarrow Y$ easily !!



Admittance (Y) Calculations

Note:

$$Y(-\ell) = \frac{1}{Z(-\ell)} = \frac{1}{Z_0} \left(\frac{1 - \Gamma(-\ell)}{1 + \Gamma(-\ell)} \right)$$

$$= Y_0 \left(\frac{1 + (-\Gamma(-\ell))}{1 - (-\Gamma(-\ell))} \right) \quad Y_0 = \frac{1}{Z_0}$$

$$\Rightarrow Y_n(-\ell) = \frac{Y(-\ell)}{Y_0} = \left(\frac{1 + (-\Gamma(-\ell))}{1 - (-\Gamma(-\ell))} \right) = G_n(-\ell) + jB_n(-\ell)$$

Define: $\Gamma' = -\Gamma$

$$Y_n(-\ell) = \left(\frac{1 + \Gamma'}{1 - \Gamma'} \right)$$

Conclusion: The same Smith chart can be used as an admittance calculator.

Same mathematical form as for Z_n :

$$Z_n(-\ell) = \left(\frac{1 + \Gamma}{1 - \Gamma} \right)$$

Admittance (Y) Chart

As an **alternative**, we can continue to use the **original Γ plane**, and add admittance curves to the chart.

$$Y_n(-\ell) = \left(\frac{1 + (-\Gamma(-\ell))}{1 - (-\Gamma(-\ell))} \right) = G_n(-\ell) + jB_n(-\ell)$$

Compare with previous Smith chart derivation, which started with this equation:

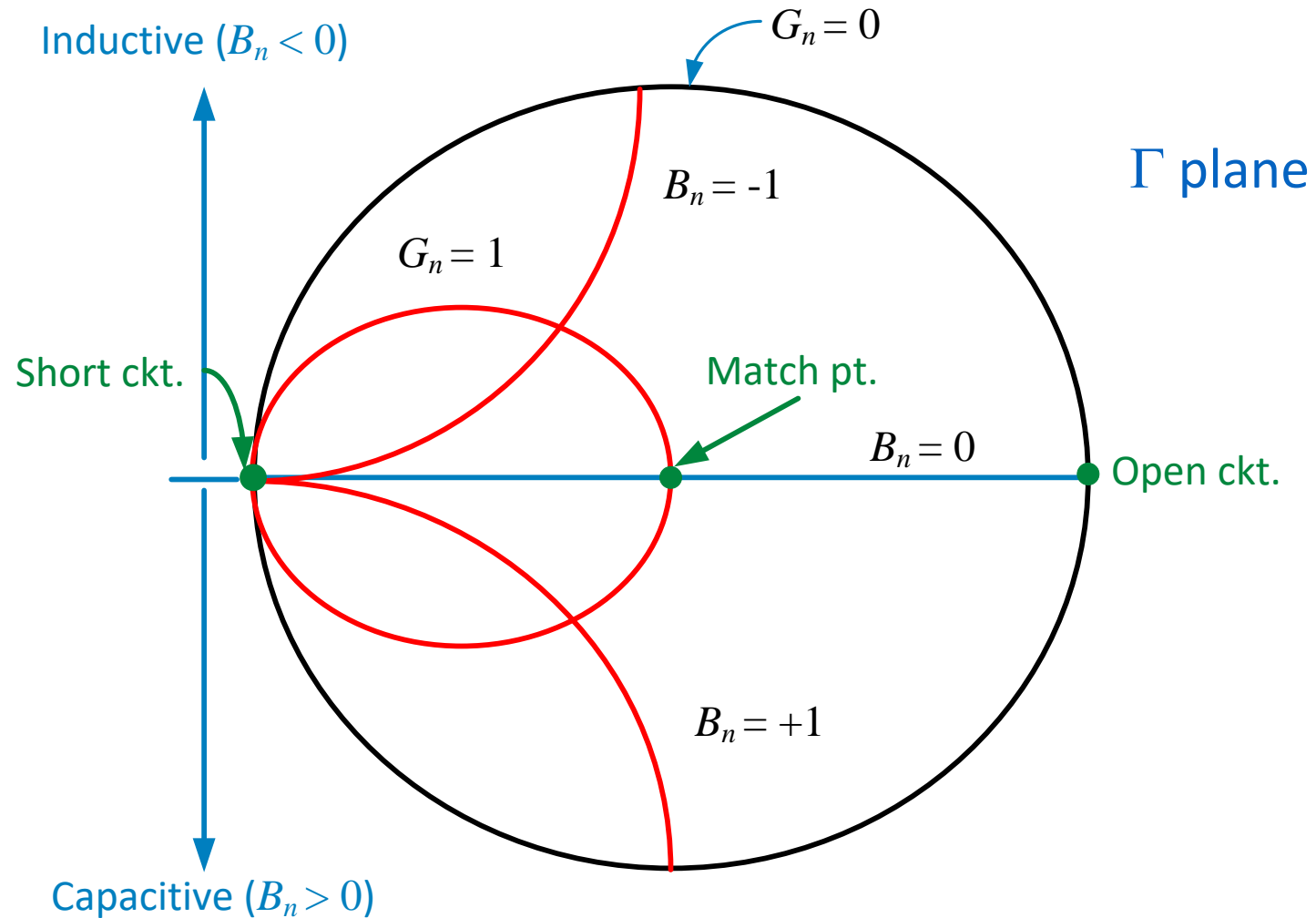
$$Z_n(-\ell) = \left(\frac{1 + (\Gamma(-\ell))}{1 - (\Gamma(-\ell))} \right) = R_n(-\ell) + jX_n(-\ell)$$

If $(R_n X_n) = (a, b)$ is some point on the Smith chart corresponding to $\Gamma = \Gamma_0$,
Then $(G_n B_n) = (a, b)$ corresponds to a point located at $\Gamma = -\Gamma_0$ (**180° rotation**).

$\Rightarrow R_n = a$ circle, rotated 180°, becomes $G_n = a$ circle.
and $X_n = b$ circle, rotated 180°, becomes $B_n = b$ circle.

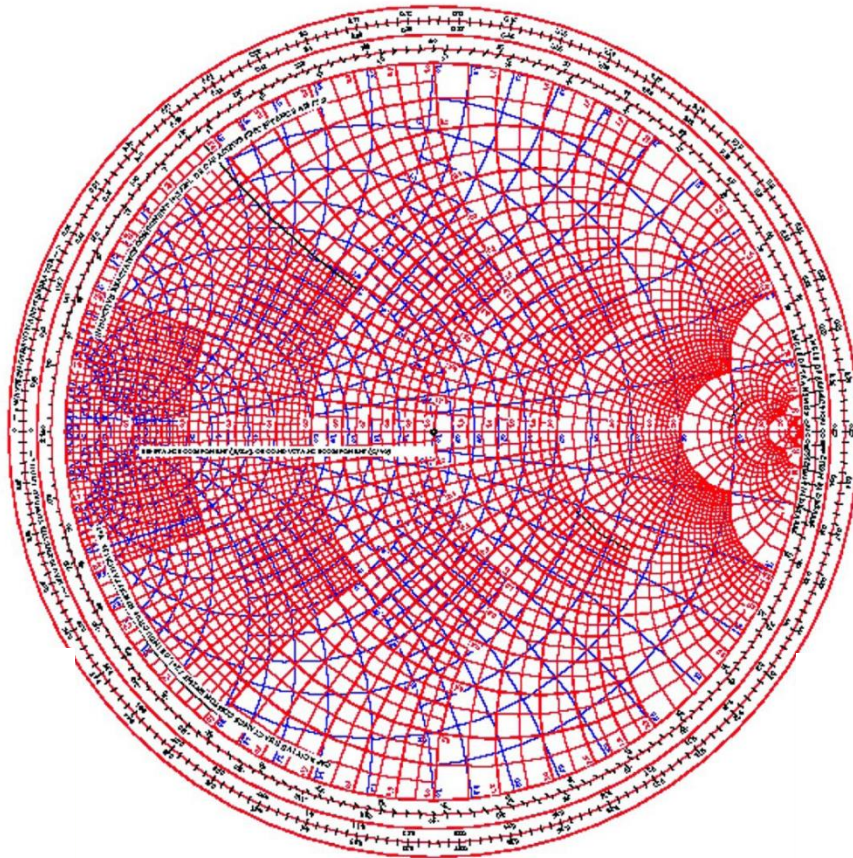
Side note: A 180° rotation on a Smith chart makes a normalized impedance become its reciprocal.

Admittance (Y) Chart (cont.)

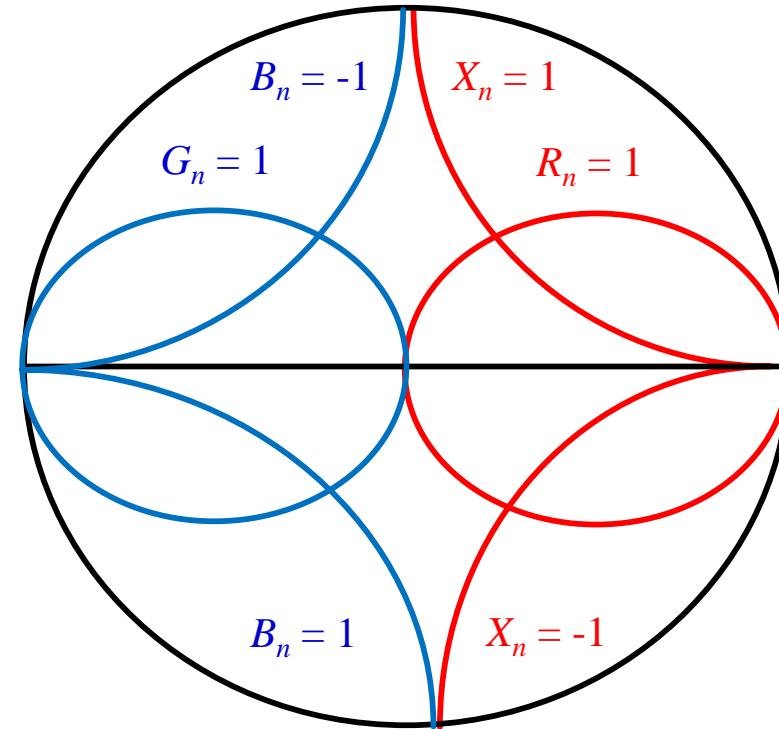


Impedance and Admittance (ZY) Chart

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



Short-hand version



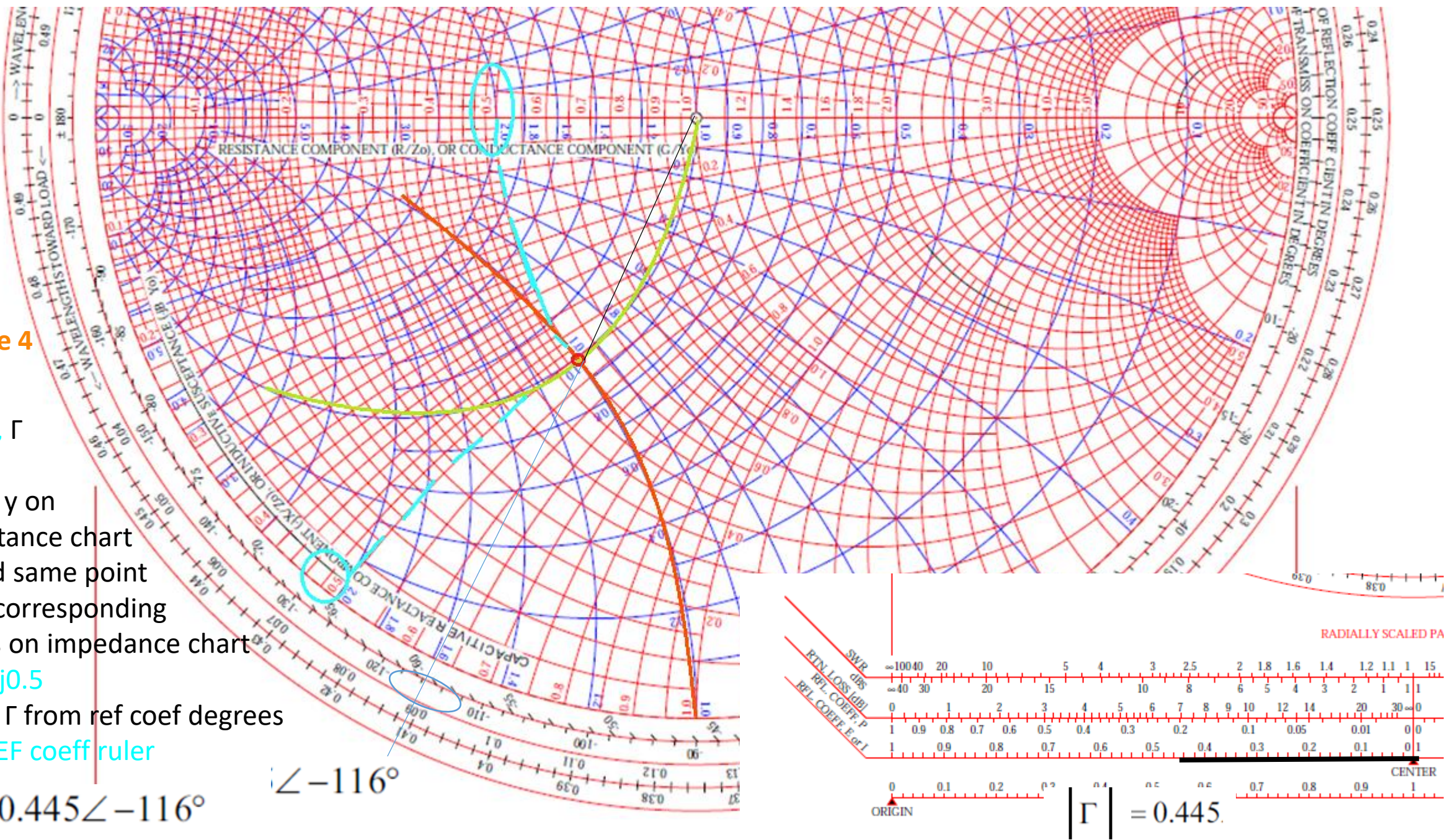
Γ plane

Example 4

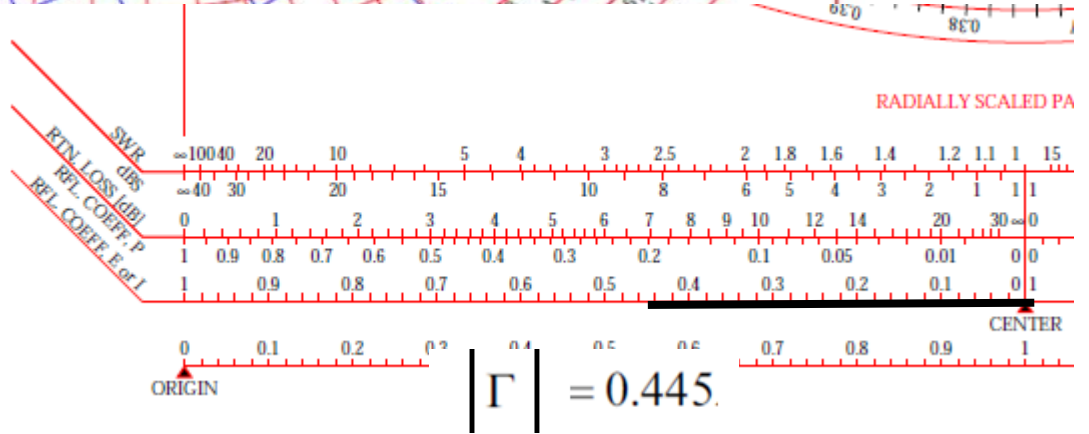
$Y=1+j$
Find Z, Γ

1. Find y on Admittance chart
2. Read same point From corresponding Circles on impedance chart
3. Find Γ from ref coef degrees And REF coeff ruler

$\Gamma = 0.445 \angle -116^\circ$



$\angle -116^\circ$



$|\Gamma| = 0.445$

Using Impedance-Admittance Smith Chart

Example 5

Given:

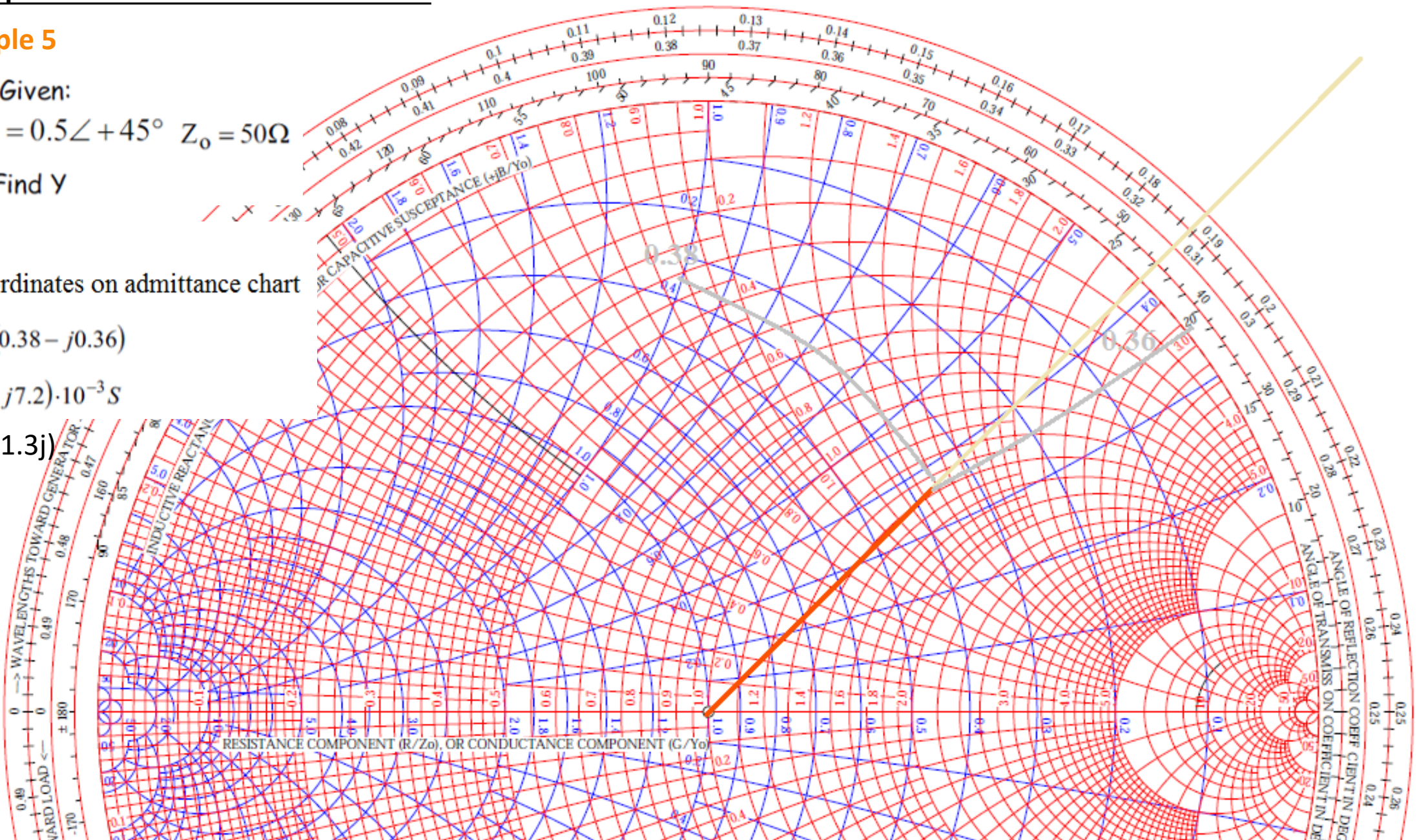
$$\Gamma = 0.5 \angle +45^\circ \quad Z_0 = 50 \Omega$$

Find Y

- Plot Γ
- Read coordinates on admittance chart

$$Y = \frac{1}{50 \Omega} (0.38 - j0.36)$$
$$= (7.6 - j7.2) \cdot 10^{-3} \text{ S}$$

$$Z = 50 * (1.38 + 1.3j)$$



Adding Elements

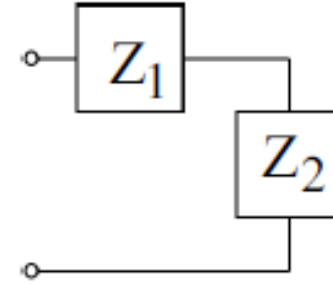
Admittance

A matching network is going to be a combination of elements connected in series AND parallel.

Impedance is well suited when working with series configurations. For example:

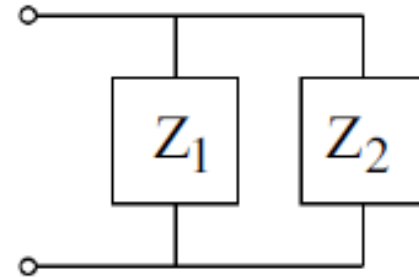
$$V = ZI$$

$$Z_L = Z_1 + Z_2$$



Impedance is NOT well suited when working with parallel configurations.

$$Z_L = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

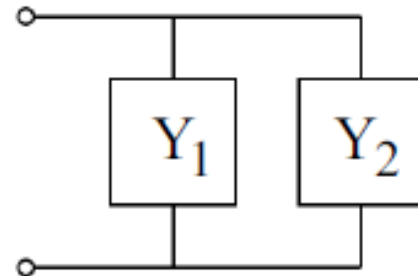


For parallel loads it is better to work with admittance.

$$I = YV$$

$$Y_1 = \frac{1}{Z_1}$$

$$Y_L = Y_1 + Y_2$$



General Rules

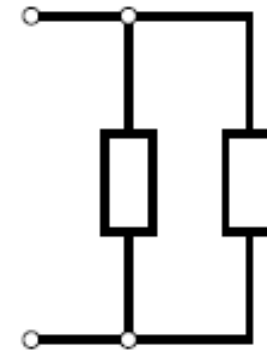
Adding Series Components

→ Impedance Smith Chart

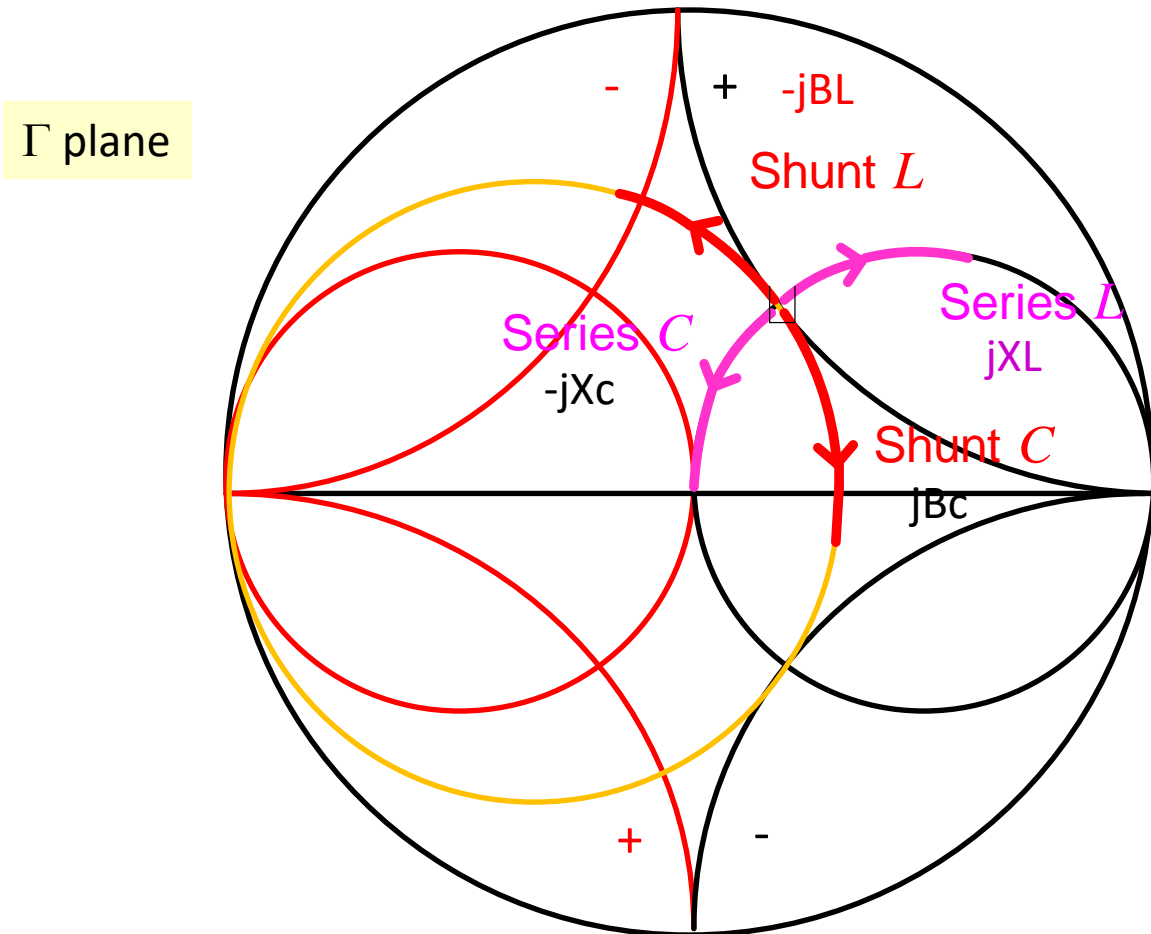


Adding Parallel (Shunt) Components

→ Admittance Smith Chart

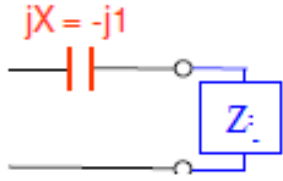


Series and Shunt Elements



Note: The Smith chart is not actually being used as a transmission-line calculator but an impedance/admittance calculator. Hence, the normalizing impedance is arbitrary.

Adding a Series Capacitor



$$Z = 0.5 + j0.7$$

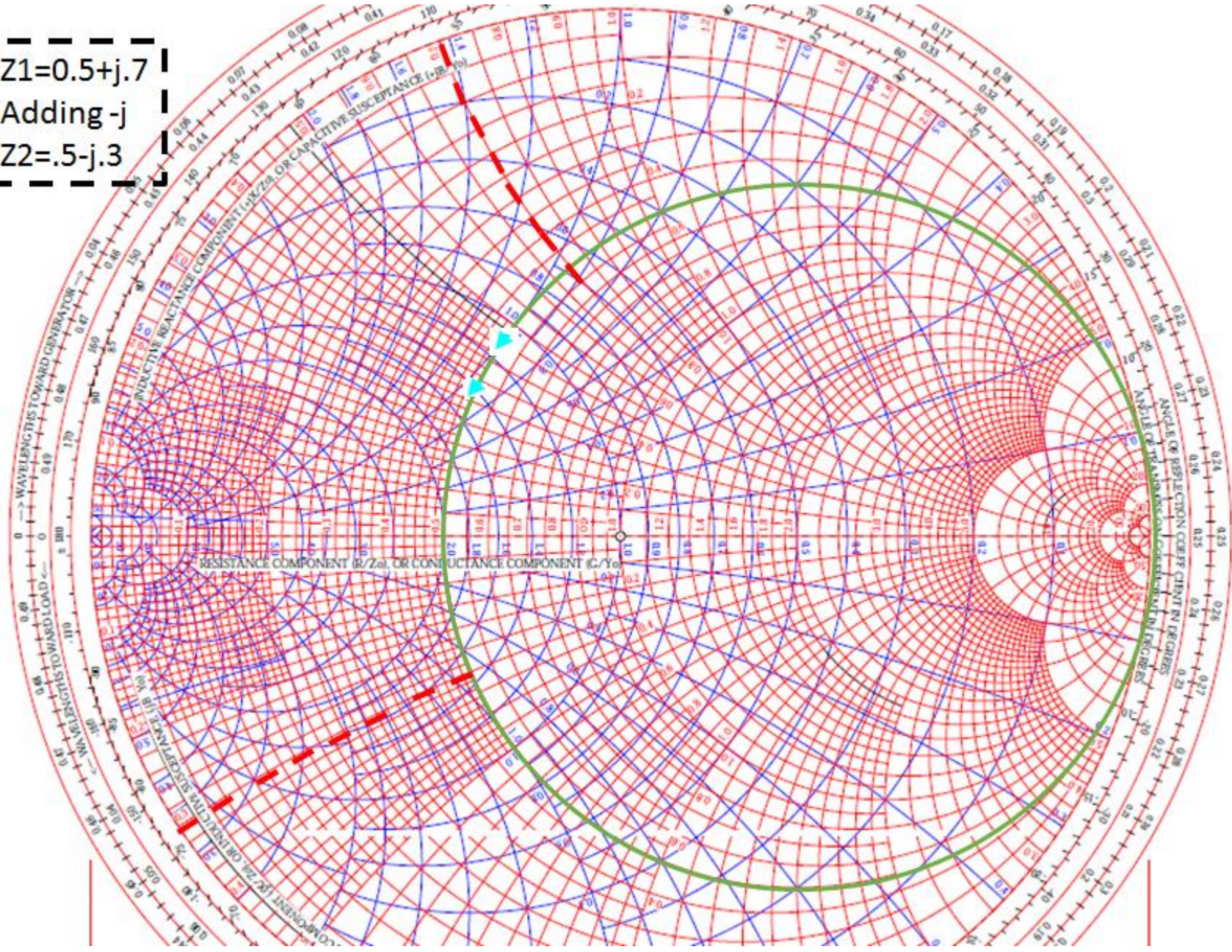
$Z_1 = 0.5 + j.7$
Adding $-j$
 $Z_2 = 0.5 - j.3$

If we have initial impedance
 $Z = 0.5 + j0.7$
We add a series capacitor

Since resistance does not change
We move on constant circle from
 $j0.7$ to $-j0.3$

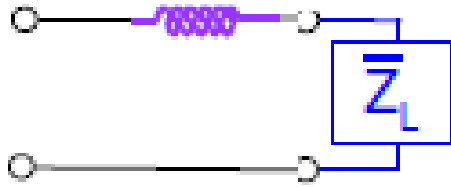
$$Z = 0.5 - j0.3$$

Values on ADS for $f = 1\text{GHz}$
 $Z = 25 + j35$
 $Z_{in} = 25 - j15$
 $1/\omega C = Z_{in} - Z = 50$
so $C = 1/(50 * 2\pi) = 3.18\text{pF}$



Adding a Series Inductor

$$jX = j1.4 = j\omega L/Z_c$$



If we have $Z=0.5-j0.4$
We add series inductor
 $jX=j1.4$

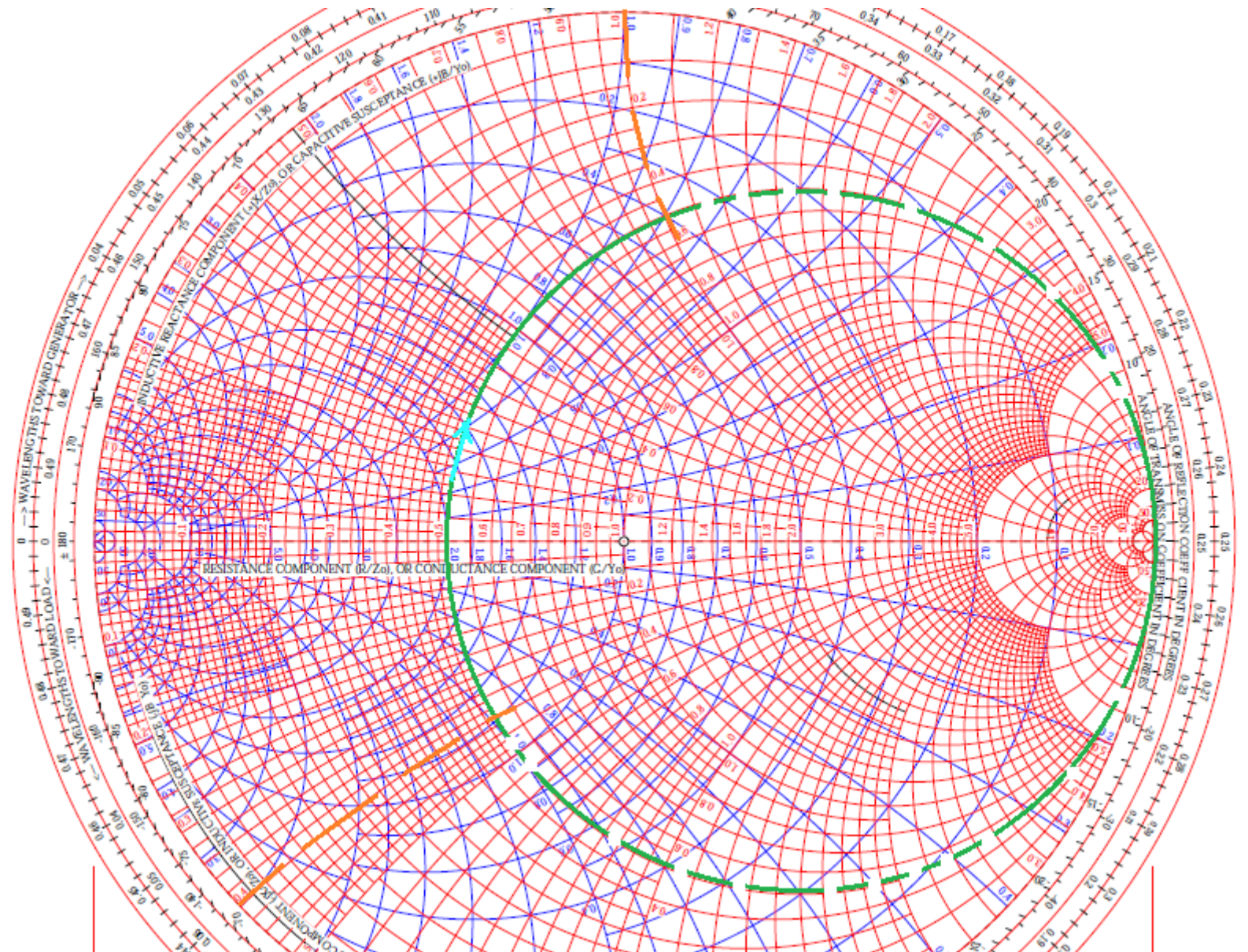
We move on resistance circle
 $Z_{in}=0.5+j1.0$

Values on ADS

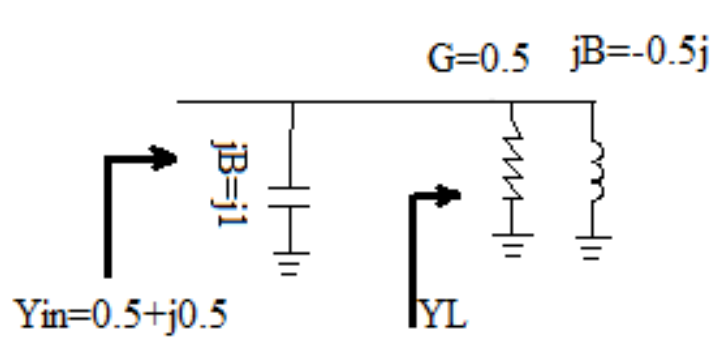
$$Z=25-j20$$

$$Z_{in}=25+j50$$

$$j\omega L=j70 \text{ so } L=70/(2\pi)=11.14\text{nH}$$



Adding a Shunt Capacitor



For $Z_L = 1 + j1.0$

On admittance chart

$Y_L = 0.5 - j0.5$

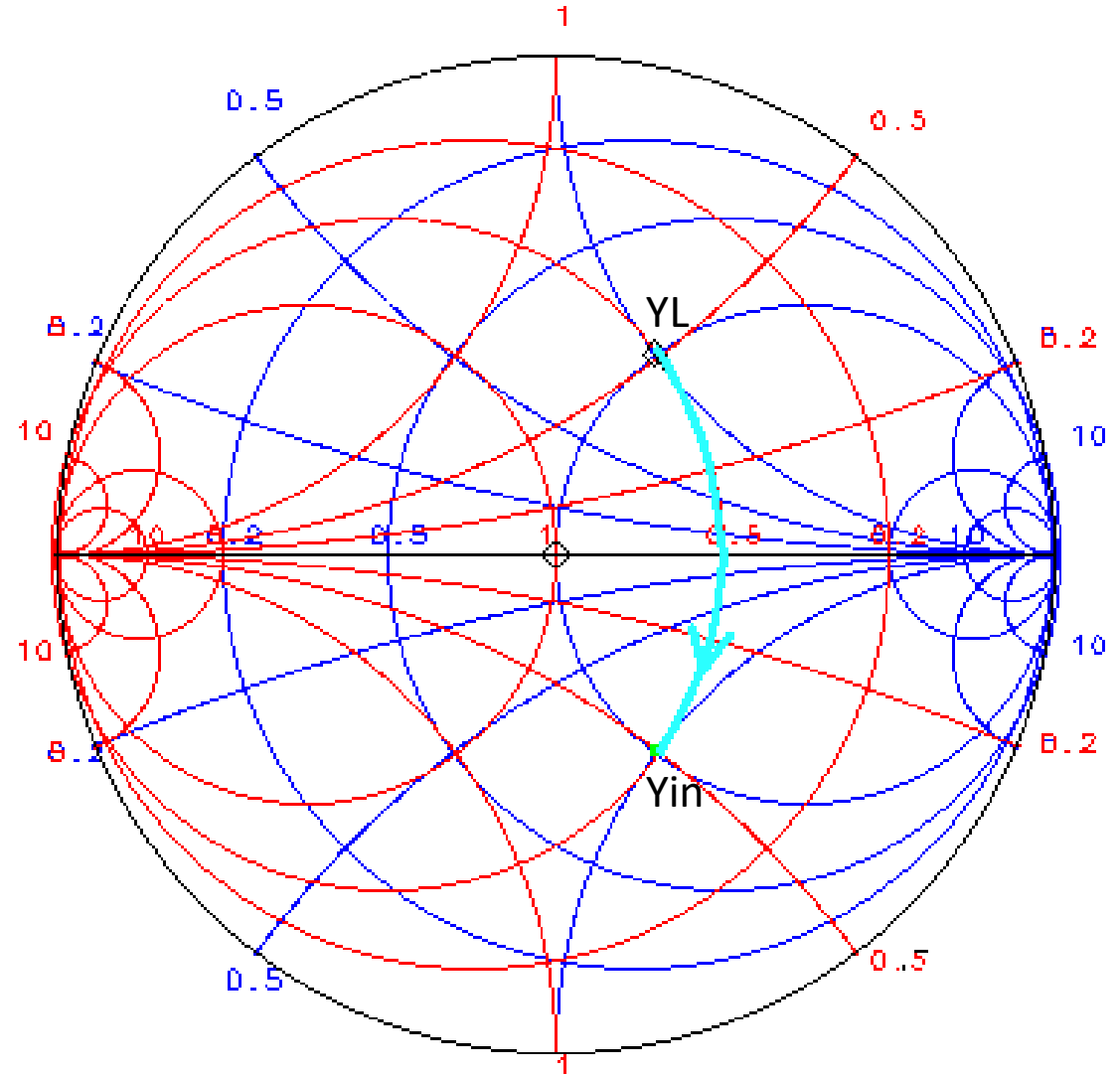
Adding shunt capacitor

With $jB = j1$

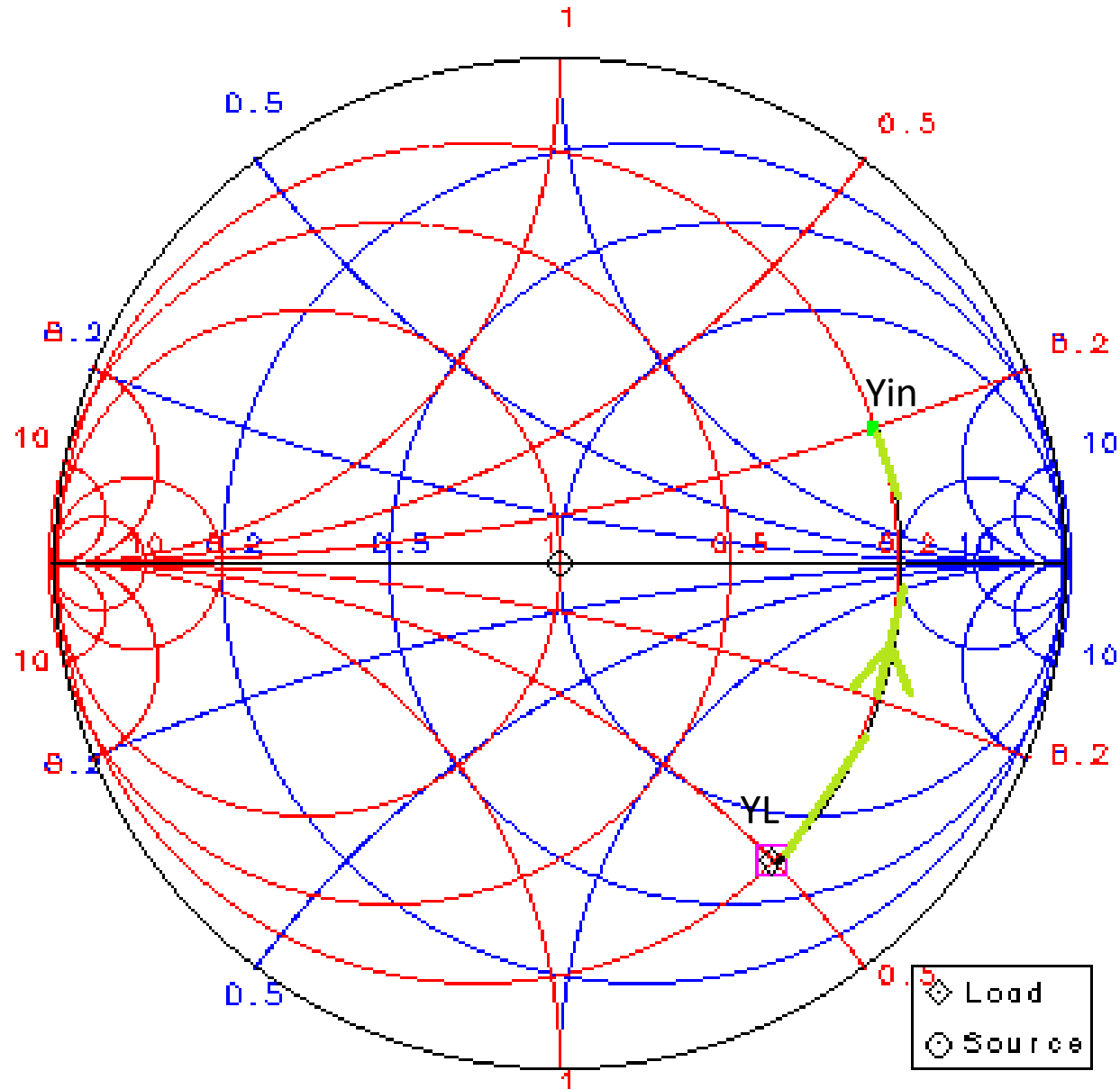
$Y_{in} = 0.5 + j0.5$

Read from impedance chart

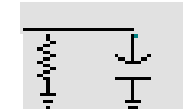
$Z_{in} = 1 - j1.0$



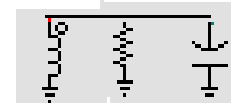
Adding Shunt Inductor



$$Y_L = 0.2 + j0.5$$



adding shunt inductor
with $JB = -j0.7$

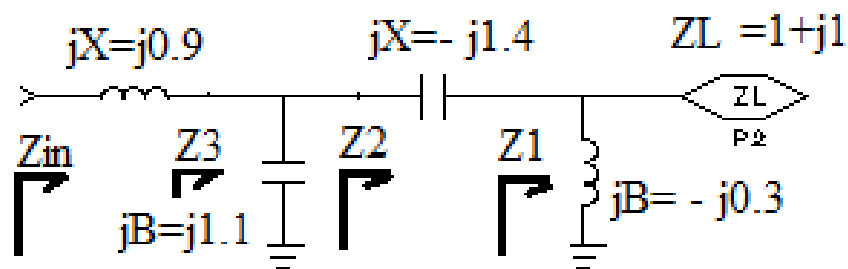


$$Y_{in} = 0.2 - j0.2$$

Read from impedance chart
 $Z_{in} = 2.4 + j2.5$

Example 6

what is the input impedance of network shown in Fig below



Solution :use impedance admittance smith chart

ZL			
Z:	1.00000	+j	1.00000
Y:	0.50000	+j	-0.50000
Z1,Y1			
Z:	0.55699	+j	0.89652
Y:	0.50000	+j	-0.80479
Z2,Y2			
Z:	0.55699	+j	-0.48811
Y:	1.01549	+j	0.88992
Z3,Y3			
Z:	0.20511	+j	-0.39989
Y:	1.01549	+j	1.97981
Zin,Yin			
Z:	0.20485	+j	0.49905
Y:	0.70391	+j	-1.71486

